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Parametric vibrations of a system containing a rectangular cross-section rotating shaft with impacts

KRYSTIAN POLCZYŃSKI[1A], GRZEGORZ KUDRA[1B*], KRZYSZTOF WITKOWSKI[1C], JAN AWREJCEWICZ[1D]

1. Lodz University of Technology, Department of Automation, Biomechanics, and Mechatronics, Łódź, Poland
[A][0000-0002-1177-6109], [B][0000-0003-0209-4664], [C][0000-0003-1214-0708], [D][0000-0003-0387-921X]
* Presenting Author

Abstract: The work is a part of a larger project concerning investigations of different configurations of connected oscillators with direct and parametric forcing sources. Among others we present preliminary investigations of dynamics of a rotating shaft system with a rectangular cross-section and a disc mounted at the end of the shaft. The disk is located in a pipe whose walls are in some distance from it. During parametric vibrations of the shaft, the disk hits the walls of the pipe. Based on the derived mathematical equations of the parametric system with impacts, a bifurcation plot was calculated for different shaft speeds. Bifurcation analysis shows both the ranges of periodic and chaotic motion.

Keywords: parametric oscillations, rectangular cross-section shaft, impact, bifurcation

1. Introduction

The dynamics of parametric systems is characterized by the existence of many stable and unstable regions. Examples of such systems where parametric vibrations occur are, for example, shafts with non-circular cross-section or with asymmetrical stiffness in different directions of vibration of such a shaft. The nonlinear behavior of these types of systems has been studied and the theory is still being developed [1–4]. In the presented work, we decided to investigate the system of connected oscillators with different forcing sources, from direct excitations to parametric forcing. We also present some preliminary research of single parametric module, i.e. a rotating massless cantilever shaft system with a rectangular cross-section and a disk at the tip (see Fig. 1a), which is assumed to be a material point. The disc is placed in a pipe so that it can impact its walls when vibrated, the gap between the disc and the pipe wall at rest is $d = 20mm$. The system is described by two conjugate differential equations

$$m_s\ddot{x}_1 + c_1\dot{x}_1 + k_{sx}x_1 + k_{sxy}y_1 + F_{Ix} = 0;\; m_s\ddot{y}_1 + c_1\dot{y}_1 + k_{sy}y_1 + k_{sxy}x_1 + F_{Iy} = -msg, \tag{1}$$

where m_s is mass of the shaft, c_1 is the viscous damping factor, $k_{sx} = \frac{3E}{l^3}\left(\frac{1}{2}\left(\frac{a^3b}{12}+\frac{ab^3}{12}\right)+\frac{1}{2}\left(-\frac{a^3b}{12}+\frac{ab^3}{12}\right)\cos(2\omega t)\right)$,

$k_{sy} = \frac{3E}{l^3}\left(\frac{1}{2}\left(\frac{a^3b}{12}+\frac{ab^3}{12}\right)+\frac{1}{2}\left(\frac{a^3b}{12}-\frac{ab^3}{12}\right)\cos(2\omega t)\right)$, $k_{sxy} = \frac{3E}{2l^3}\left(-\frac{a^3b}{12}+\frac{ab^3}{12}\right)\sin(2\omega t)$ are stiffness coefficients of the shaft with a, b dimensions of the cross section and Young modulus E, msg is the moment of gravitational force, l is a length of the shaft, and finally F_{Ix}, F_{Iy} are forces come from impact between disc and pipe's wall. The impact forces are defined as

$$F_{Ii}(x_1, y_1, \dot{x}_1, \dot{y}_1) = \left. \frac{i_1 F_I\left(\sqrt{x_1^2 + y_1^2}, \frac{\dot{x}_1 x_1 + \dot{y}_1 y_1}{\sqrt{x_1^2 + y_1^2}}\right)}{\sqrt{x_1^2 + y_1^2}} \right|_{i = x_1, y_1}; \quad \begin{matrix} f(\alpha, \beta) = (b_I \beta + 1)|x - d|^{3/2}; \\ F_I(\alpha, \beta) = k_I f(\alpha, \beta) \mathbf{1}[\alpha - d] \mathbf{1}[f(\alpha, \beta)]. \end{matrix} \tag{2}$$

Terms b_I, k_I are constant parameters of obstacle stiffness and obstacle damping, respectively.

2. Results and Discussion

In Fig. 1b we present a bifurcation analysis of a single parametric forcing module for the angular velocity of a rotating shaft as a control parameter. It is easy to see that the dynamics of the system is varied, both periodic and chaotic motion windows are visible. The pipe limiting shaft deflections prevents the system from escaping to infinity. Instead it appears a complex chaotic dynamics with impacts.

(a) (b)

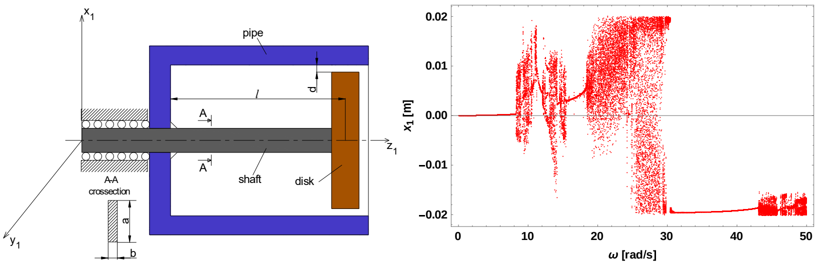

Fig. 1. Scheme of a prismatic rotating shaft with impacts (a) and bifurcation diagram (b)

3. Concluding Remarks

The investigations concern parametric vibrations of a system of connected mechanical oscillators with different kinds of excitations. Preliminary studies show that even the dynamics of a single parametric excitation module in the form of a rotating shaft with a rectangular cross-section with a rigid limiter of motion shows complex bifurcation and chaotic dynamics. The studied problem may occur in electric motors but also in other machines. The examination of the problem might help to eliminate the harmful phenomena caused by impacts in such systems.

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