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The dynamics of two coupled oscillators with the same damping term

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Abstract: In this paper we present the dynamic behavior of a system of linear and nonlinear coupled oscillators with the same damping term implemented by a nonlinear electronic circuit. The dynamic behavior is directly related to the damping parameter as well as the other parameters. We observe the alternation between periodic, chaotic and quasi-periodic oscillation.

Keywords: nonlinear oscillator, chaotic behavior, electronic circuits

1. Introduction

Systems of coupled oscillators play an essential role in physics and engineering [1]. In this work, we study a system of linear and nonlinear coupled oscillators with the same damping term. The system is mathematically described by

$$\begin{aligned}\dot{x} &= p_x \\ \dot{p}_x &= -k_1 x^3 - \lambda p_x - \alpha x y \\ \dot{y} &= p_y \\ \dot{p}_y &= -k_2 y - \lambda p_x\end{aligned} \tag{1}$$

k_1 and k_2 are the nonlinear and the linear coefficients, α is a coupling parameter and λ the damping parameter. The system has only one non-hyperbolic equilibrium point (0,0,0,0), so the possible attractors are hidden [2]. The electronic circuit of figure 1 implements the above nonlinear system.

2. Dynamic behavior

Figure 2 presents the bifurcation diagram[3] of the nonlinear variable x related to the damping parameter λ for $k_1 = 1, k_2 = 1, \alpha = 1.5$ and values of the initial conditions $x(0) = 5, p_x(0) = 0.001, y(0) = 1, p_y(0) = 0.001$. The system oscillates in different ways: regularly with variant periods, quasi-periodically, and chaotically related to the damping parameter. The maximal Lyapunov Characteristic Exponent[4] of the system confirms this rich dynamical behavior.

Figure 4 presents the behaviour of the nonlinear variable x related to the coupling parameter α for $k_1 = 1, k_2 = 1, \lambda = 0.2$ and the same initial conditions . For values of $\alpha < 0.2$ the system oscillates periodically and then for $\alpha \geq 0.2$ starts to oscillate chaotically. Also, for $\alpha > 1.2$ there is a change between regular and non regular oscillations.

3. Discussion

The addition of the same damping parameter of the nonlinear oscillator in an undamped oscillator drastically changes the dynamics of the system of the coupled oscillators and leads to a rich dynamical behavior. Both the damping and the coupling parameter play an essential role in the oscillations of the system.

The nonlinear electronic circuit that implements the proposed system allows us to study and experimentally confirm the above results. Also, it opens perspectives for using such oscillators in electronic applications.

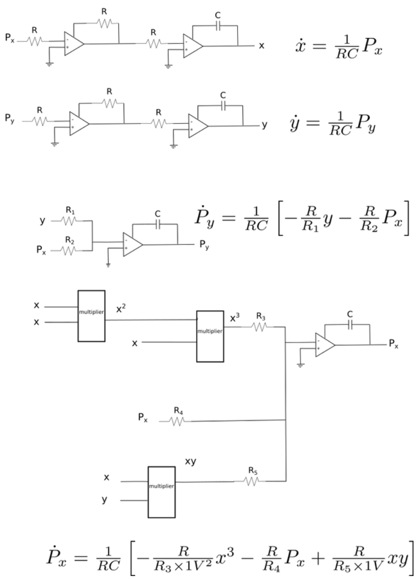

Fig.1: Circuit implementation

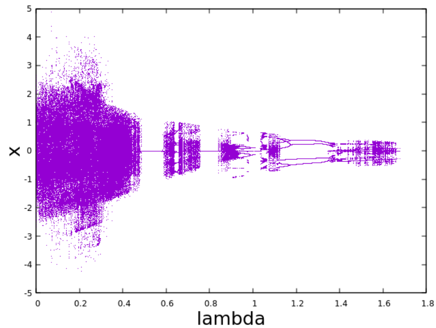

Fig.2: Bifurcation diagram.

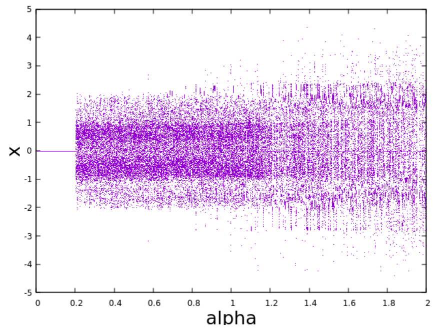

Fig.3: The behaviour of the nonlinear variable related to thecoupling parameter.

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