2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Chaos in Thirring Model

EREN TOSYALI[1*], FATMA AYDOGMUS[2]

1. Istanbul Bilgi University, Vocational School of Health Services, [0000-0001-9118-851X]
2. Istanbul University, Faculty of Science, Physics Department [0000-0003-1434-2143]
* Presenting Author

Abstract: Thirring model is a two-dimensional, conformal invariant spinor model that plays an important role as a test model in the Quantum Field Theory. Thirring instantons were found using the method of spontaneously broken conformal symmetry in the Thirring nonlinear spinor field equation. In this study, we investigate the dynamic of Thirring instantons under forcing and damping by constructing Poincare sections. Thirring instantons exhibit regular and chaotic behaviours depending on system parameters.

Keywords: Dynamic, Instanton, Thirring, Chaos.

1. Introduction

Instantons, as a special type of solitons that propagate without losing their shape and speed properties and can maintain their unique properties during any collision, are solutions with zero energy and finite action with space-time expansion [1]. Instantons, which play an important role in theoretical and mathematical physics and correspond to the vacuum state, have space-time expansion. Spinor-type instanton solutions were found by the spontaneously broken conformal symmetry in nonlinear Thirring field equation system [2-3]. Thirring nonlinear differential equation system is given as:

$$2\frac{dF(u)}{du}+\frac{1}{2}F(u)-\alpha AB(F(u)^2+G(u)^2)G(u)=0 \quad (1)$$

$$2\frac{dG(u)}{du}-\frac{1}{2}G(u)+\alpha AB(F(u)^2+G(u)^2)F(u)=0 \quad (2)$$

We present Thirring field equation system with forcing and damping:

$$2\frac{dF(u)}{du}+\frac{1}{2}F(u)-\beta(F(u)^2+G(u)^2)G(u)=0 \quad (3)$$

$$2\frac{dG(u)}{du}-\frac{1}{2}G(u)+\beta(F(u)^2+G(u)^2)F(u)-A_T Cos(w_T H(u))+\gamma G(u)=0 \quad (4)$$

$$\frac{dH(u)}{du}=0 \quad (5)$$

2. Results and Discussion

In our previous works, the role of the coupling constant in the evolution of Thirring instantons in phase space has been studied via Heisenberg ansatz [4-5]. Thirring instantons have a Duffing oscillator type characterization without forcing and damping as stated in the Figure 1.

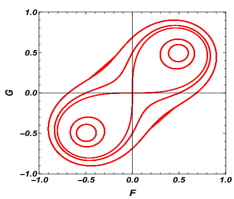

Fig. 1. Duffing oscillator type characterization of Thirring Instantons

In this paper, we investigate the dynamic of nonlinear Thirring instantons by changing the forcing and damping parameters (γ, A and ω) to get more information on spinor type Thirring instantons. System parameters for phase space and Poincare sections are taken as ω = 0.57, γ = 0.05 and initial conditions are (F (0), G(0) = (0.158, 0.55)). The differential equations solved with step size 0.1 and lengh 6000.

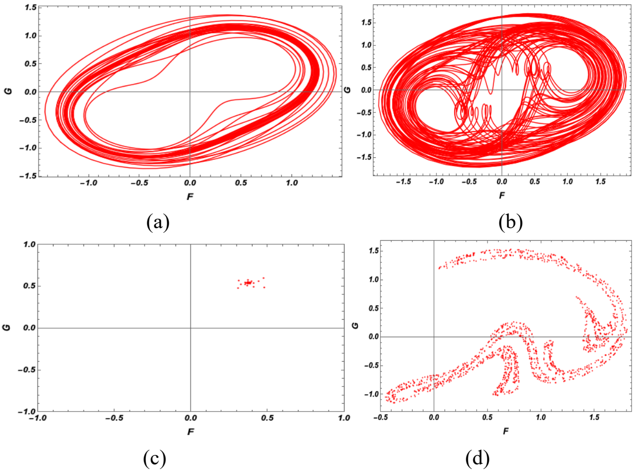

Fig. 2. Phase space diagrams of forced and damped Thirring instantons at ω = 0.57, (a) A = 0.25, (b) A = 1.2 and Poincare sections (c) A = 0.25, (d) A = 1.2 .

3. Concluding Remarks

The regular and chaotic instanton solutions of forced and damped Thirring model are studied in phase space. The obtained phase space diagrams and the Poincare sections show that Thirring instantons lost their stability in phase space under forced and damped terms. Forcing and damping terms destroy the regularity and forms a chaotic layer.

Acknowledgment: This work was supported by the Scientific Research Projects Coordination Unit of Istanbul University; Project no: FBA-2018-28954.

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