16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line
dys-ta.com

Influence of Nonlinear Flexural and Sway Interaction on the Global Dynamics of an Axially Loaded Column

DIEGO ORLANDO[1], PAULO B. GONÇALVES[2*], JULIANA M. P. RAIMUNDO[2]

1. State University of Rio de Janeiro, Resende, Brazil [0000-0002-2702-7970]
2. Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil [0000-0001-8645-3542]

Abstract: In the present work, the global dynamics of a 2dof conceptual column model that exhibits nonlinear modal interaction between the flexural and sway modes is studied. The results show that its buckling load can be dominated by the flexural or the sway mode, and, in the latter case, strong modal coupling is observed, leading to several post-critical paths, most of them unstable for load levels lower than the buckling load, thus limiting the structures load carrying capacity. Also the structure is sensitive to initial disturbances and dynamic loads that may lead to escape from the safe pre-buckling potential well. Here the dynamic behavior of the model under base excitation is explored. It is shown through the analysis of bifurcations diagrams and basins of attraction that the safety of the model is profoundly affected by the erosion and stratification of the basins of attraction and imperfections.

Keywords: Modal interaction, unstable buckling, global dynamics, basin erosion, dynamic integrity.

1. Introduction

In slender frames the buckling and post-critical behavior is affected by lateral constraints, being the unconstrained buckling load much lower than the constrained one. Depending on their geometry, primary and secondary bifurcations may result in multiple equilibrium configurations due to nonlinear modal interaction, depending on the relation between flexural and lateral stiffness. The existence of multiple equilibrium positions delimits the set of initial conditions that converge to the desired pre-buckling solution. This problem can only be tackled using the tools of global nonlinear dynamics. In particular, the analysis of the evolution and erosion of the basin of attraction of a desired solution has been shown to be a key issue in attaining a safe design [2]. These phenomena are here explored using a 2dof phenomenological model proposed by Raithel and Clemente [1] and investigated in [2].

2. Results and Discussion

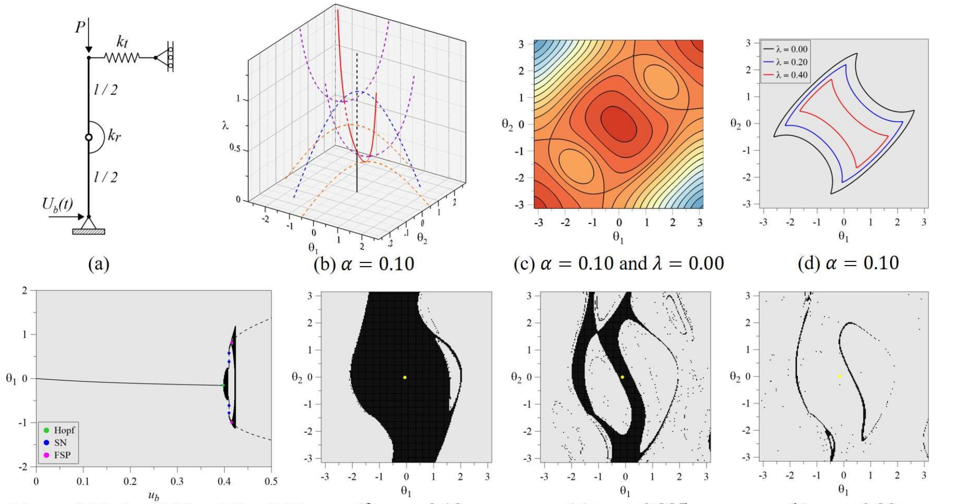
The model is composed of two rigid bars of length $l/2$, with mass per-unit length m, connected by a rotational spring of stiffness k_r and supported at the top by a lateral spring of stiffness k_t, Figure 1(a). P is the static axial load, $U_b(t) = U_b \sin(\Omega t)$ is the base excitation and θ_1 and θ_2 are chosen as the two degrees of freedom. The equations of motion are given in nondimensional form by:

$$\begin{aligned}\frac{1}{6}\ddot{\theta}_1 &+ \frac{1}{16}(cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2)\ddot{\theta}_2 - \frac{1}{16}(cos\theta_1 sin\theta_2)(\dot{\theta}_2)^2 + \frac{1}{16}(sin\theta_1 cos\theta_2)(\dot{\theta}_2)^2 \\ &- 2\xi_1(\dot{\theta}_2 - \dot{\theta}_1) + 2\xi_2(\dot{\theta}_1 cos\theta_1 + \dot{\theta}_2 cos\theta_2)cos\theta_1 - \alpha(\theta_2 - \theta_1) \\ &+ \frac{1}{4}(sin\theta_1 + sin\theta_2)cos\theta_1 = \frac{1}{2}\lambda\, sin\theta_1 + \frac{3}{8}u_b\, \beta^2 sin(\beta\tau)cos\theta_1\end{aligned} \quad (1)$$

$$\begin{aligned}\frac{1}{24}\ddot{\theta}_2 &+ \frac{1}{16}(cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2)\ddot{\theta}_1 + \frac{1}{16}(cos\theta_1 sin\theta_2)(\dot{\theta}_1)^2 - \frac{1}{16}(sin\theta_1 cos\theta_2)(\dot{\theta}_1)^2 \\ &+ 2\xi_1(\dot{\theta}_2 - \dot{\theta}_1) + 2\xi_2(\dot{\theta}_1 cos\theta_1 + \dot{\theta}_2 cos\theta_2)cos\theta_2 - \alpha(\theta_2 - \theta_1) \\ &+ \frac{1}{4}(sin\theta_1 + sin\theta_2)cos\theta_2 = \frac{1}{2}\lambda\, sin\theta_2 + \frac{1}{8}u_b\, \beta^2 sin(\beta\tau)cos\theta_2\end{aligned} \quad (2)$$

where the dots represent the derivatives with respect to the nondimensional time $\tau = \omega t$, $\alpha = k_r/k_t l^2$, $\lambda = P/k_t l$, $\omega^2 = k_t l^2/A\rho l^3$, $2\omega\xi_1 = C_1/A\rho l^3$, $2\omega\xi_2 = C_2 l^2/4A\rho l^3$, $u_b = U_b/l$, and $\beta = \Omega/\omega$, is the ratio of the excitation frequency (Ω) and lowest natural frequency (ω).

Figure 1(b) shows the equilibrium paths for $\alpha = 0.10$. The fundamental equilibrium path is stable up to the critical load (λ_{cr}). There are two post-critical paths: an ascending path at –45° associated with $\lambda_{cr1} = 4\alpha$ (in red) and an unstable descending path at +45°, associated with $\lambda_{cr2} = 1$ (in blue). Another four coupled unstable secondary paths emerge along the ascending post-critical path. Thus, for $\lambda < \lambda_{cr}$ the prebuckling solution is surrounded by four saddles and two maxima, which delimits the safe static basin, Fig. 1(c). This safe region decreases with the compressive load λ, Fig. 1(d). The bifurcation diagram for $\alpha = 0.20$, $\lambda = 0.20$ and $\beta = 2.05$, Fig. 1(e), and the evolution of the basins of attraction with increasing values of the u_b, Figs. 1(f) – (h), illustrates the marked erosion of the basins of attraction and loss of dynamic integrity.



(a) (b) $\alpha = 0.10$ (c) $\alpha = 0.10$ and $\lambda = 0.00$ (d) $\alpha = 0.10$

(e) $\alpha = 0.20$, $\lambda = 0.20$ and $\beta = 2.05$ (f) $u_b = 0.10$ (g) $u_b = 0.235$ (h) $u_b = 0.30$

Fig. 1. (a) Discrete model [1]; (b) Fundamental solution and the post-critical paths; (c) and (d) Energy surfaces; (e) Bifurcation diagram; (f), (g) and (h) Basins of attraction. $\xi_1 = \xi_2 = 0.01$.

3. Concluding Remarks

In this work the influence of the nonlinear interaction between flexural and sway buckling is studied through a detailed parametric analysis, a problem of importance in structural stability and nonlinear dynamics. The inherent strong modal coupling leads to various unstable post-buckling solutions that control the geometry of the safe pre-buckling potential well and, consequently, the global behavior of the system under dynamic loads, as shown briefly by these novel results.

Acknowledgment: Brazilian research agencies CAPES, CNPq and FAPERJ.

References

[1] RAITHEL A, CLEMENTE P: On the stability of pin-joined stiffened frames. *Thin-Walled Structures 1994*, **19**(2-4):211-220.

[2] ORLANDO D, RAIMUNDO JMP, GONÇALVES PB: Flexural and Sway Interaction in the Nonlinear Vibrations of a Phenomenological Model. *In Nonlinear Dynamics of Structures, Systems and Devices: Proceedings of the NODYCON 2019* 2020, **I**: 207. Springer Nature.