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Real and Noice-Induced Bubbling: Geometric Approach and Problem of Initial Deviation

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Abstract: We explain the mechanisms leading to the bubbling phenomenon, i.e., high-frequency oscillations disrupting the waveform of slowly oscillating signals in a restricted phase interval.

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1. Introduction

The term bubbling refers to a phenomenon manifesting itself as high-frequency oscillations that disrupt the waveform of a slowly oscillating signal in a restricted phase interval. Although this phenomenon has been known for more than 20 years and observed (both numerically and experimentally) in all kinds of power electronic converters, no convincing explanation of the mechanism behind its occurrence has been found. It has been reported in many publications it occurs after a smooth bifurcation, such as a pitchfork or Neimark-Sacker, and therefore, it has been widely assumed that bubbling is caused by these bifurcations, although the specific mechanism leading to the onset of bubbling remained unknown. Recently, the onset of bubbling has been observed (both experimentally and numerically, see Fig 1(a) and (b), respectively) in a power electronic inverter that does not exhibit any smooth bifurcations in the relevant parameter domain.

The specific model considered in the presented work describes the behavior of the inverter system mentioned above and is given by a 1D non-autonomous piecewise smooth map (1) $x_{k+1}=f(x_k,k)$ presented in [1]. The signals of the considered inverter correspond to m-cycles of map (1), where the ratio m=200 is the ratio between the fast switching frequency and the low reference frequency.

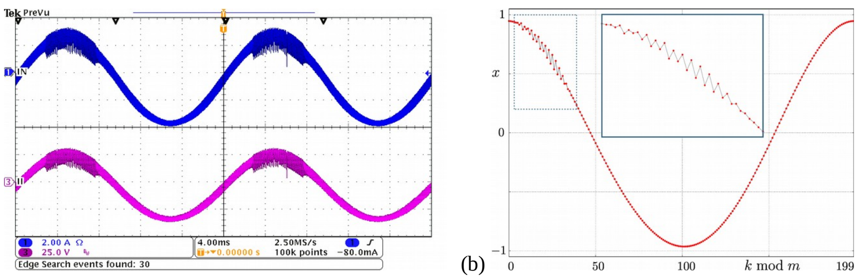
(a) (b)


Fig. 1. Bubbling-affected orbits observed (a) experimentally, (b) numerically in discrete time.

2. Results

The first significant result we obtained has been reported in [1]. We have shown that in both cases, (i.e., the onset of bubbling associated with smooth bifurcation, as previously reported, and without as recently discovered) *the mechanism leading to bubbling is of a geometric rather than a topologic nature*. In fact, we identified regions in the state space such that an orbit passing through these regions (non-bubbling intervals) at all phases is not bubbling-affected. If at some phase the point x_k leaves the non-bubbling interval, it exhibits bubbling at this phase.

Furthermore, we discovered a novel phenomenon, called *simmering*, which represents a weaker form of phase-dependent distortion of the signal and manifests itself as bubbling of the first order forward differences (derivative with respect to discrete time). For this phenomenon, similar regions (non-simmering intervals) can be defined. Moreover, the procedure can be generalized for higher-order derivatives, which leads to a sequence of the rank-i non-bubbling intervals in the state space associated with non-bubbling of the i-th derivative. Typically, at each phase, these intervals are nested into each other so that the onset of bubbling of the i-th derivative precedes – in the state space as well as in the parameter space – the onset of bubbling of the (i-1)-th derivative. This demonstrates that *the onset of bubbling is a gradual process*. For example, the onset of simmering precedes the onset of bubbling and can be used, from a practical point of view, as a kind of an early warning system.

The presented geometric approach to bubbling does not explain the reason why an orbit may leave the non-bubbling intervals in a well-defined and clearly restricted phase interval. To understand the reason, note that the behavior of the function $f(x,k)$ can be described by a set of m autonomous functions $f_k(x)$ which – depending on the phase k – may be contractive or expanding. A general property of models for various power converters is that they may have a long phase interval associated with expanding functions only, followed by a phase interval in which all functions are contractive. *A small deviation at the beginning of the expanding phase interval gets amplified by expanding functions, causing the orbit to leave non-bubbling intervals of decreasing ranks.*

We have identified several reasons that cause this initial deviation. In the case that bubbling occurs after a smooth bifurcation, e.g. a pitchfork, the normal form of this bifurcation forces the “new” orbits appearing at the bifurcation to move quickly apart from the “old” orbit which existed before. As a consequence, the “old” (unstable) orbit remains not bubbling-affected, while the “new” solutions successively leave the non-bubbling intervals of decreasing ranks and start to exhibit bubbling. Therefore, the onset of bubbling does not occur immediately at the bifurcation point, as previously assumed, but very soon after. A similar effect may be caused by a persistence border collision, which does not change the stability or the periodicity of the cycles, but may lead to bubbling.

The most unexpected phenomenon we discovered is *noise-induced bubbling*. In this case, the cycles of map (1) are not bubbling-affected, while the signals observed in experiments exhibit well-developed bubbling. Here, the deviation is caused by noise which is omnipresent in physical experiments and gets strongly amplified by expanding functions. Moreover, in numerical simulations, this kind of bubbling occurs as well. Here, the deviation is caused by numerical noise, i.e. the limited precision of floating-point numbers. So, as a curious feature, both physical experiments and numerical simulations might exhibit bubbling while the ideal solution is not bubbling-affected. In this way, map (1) can be used to predict *whether* bubbling occurs and at *which part of the phase domain*, but cannot predict *at which parameter values* unless a suitable noise model is added.

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References

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