16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Nonlinear wave propagation in one-dimensional metamaterials via Hamiltonian perturbation scheme

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Abstract: The work investigates the wavefrequencies and waveforms of a periodic waveguide with local vibration absorbers, featuring both weak geometric nonlinearities and nonlinear viscoelastic damping. The approach uses tools borrowed from Hamiltonian perturbation theory, as well as techniques classically employed in the context of nearly-integrable Hamiltonian systems in order to describe the solutions asymptotically. A dedicated analysis is oriented towards the case of weak dissipation, whilst the zero-dissipation limit case is presented for comparison with the literature. The obtained asymptotic expansions for the non-linear frequency spectra and the invariant manifolds of the system are presented. The analysis is carried out either in the non-resonant case or in the 3:1 internal resonance setting, and this is achieved by using an adaptation of the classical tools of resonant normalization techniques developed in the Hamiltonian context. Finally, the asymptotic results are also validated via numerical simulations.

Keywords: Mechanical metamaterials, Nonlinear damping, Cubic nonlinearities, Energy dissipation, Lie series

1. Introduction

Metamaterials with an engineered cellular arrangement of functionalized micromechanisms exhibiting amplitude-dependent dispersion properties are attracting increasing interest in nonlinear dynamics. Moreover, the band structure of microstructured periodic waveguides is of wide interest, especially regarding the amplitude-dependent pass and stop bands of oscillator chains and other periodic structures. Extensive efforts are currently targeted towards designing mechanical cellular metamaterials characterized by large-amplitude band gaps with tunable low-centerfrequency. To this end, intracellular mechanisms of local-global resonance can be realized by highly-flexible and damped oscillators working as propagation inhibitors and energy absorbers.

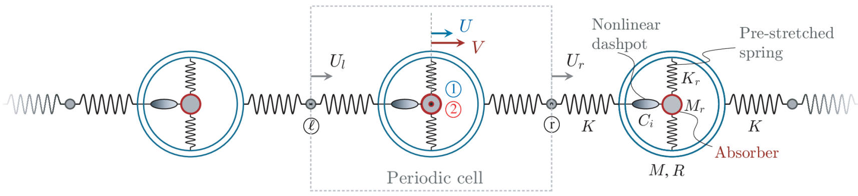

Fig. 1. Cellular metamaterial with embedded locally-resonant vibration absorbers.

2. Results and Discussion

A minimal locally resonant system can be realized as an infinite one-dimensional dissipative chain of rigid rings, embedding oscillators with geometric and material nonlinearities. The introduced nonlinear visco-elastic damping allows a better tunability of the amplitude-dependent damping towards an improved passive control performance (see Fig. 1). Denoting by u and w the displacement of the periodic diatomic cell, and the relative displavement of the resonator, respectively, the free wave propagation is governed by the following coupled system of non-linear dimensionless ODEs

$$(1+\rho^2)\ddot{u}+\rho^2\ddot{w}+2u+(1-\cos\beta)u=0$$
$$\rho^2\ddot{u}+\rho^2\ddot{w}+\xi_1(1-\xi_2)\dot{w}+\xi_1\xi_2\xi_3\dot{w}w^2+2\mu w+(\eta-\mu)w^3=0$$

where β is the wavenumber and the coefficients $\rho^2,\xi_1,\xi_2,\xi_3,\mu,\eta$ are the mechanical parameters. From the methodological viewpoint, the perturbation analysis of the governing equations is carried out in the neighbourhood of a hyperbolic equilibrium for the system after a suitable rescaling [1]. The employed technique relies on the possibility to interpret a given system of ODEs as a Hamiltonian system in a suitably extended phase space. Accordingly, the associated Hamiltonian function can be directly treated by using well established tools of Perturbation Theory for nearly-integrable Hamiltonian systems. Canonical transformations of variables generated through Lie-series operators are employed [2] while the approach here adopted differs from the commonly used perturbation techniques based on the method of multiple scales [3,5]. Under mild non-resonance hypotheses on the eigenvalues of the linearized system, the (first-order) correction to the linear frequencies arise naturally from the normal form construction. A comparison with literature results corresponding to the zero-dissipation limit [4] is discussed, together with the case of weak dissipation in which the construction of invariant manifolds is shown to be feasible. Starting from the study proposed in [1], the case of 3:1 resonance is investigated in this work by proposing an approach which relies on techniques arising from the construction of resonant normal form in the Hamiltonian setting. Either the description of the spectra, or the invariant manifolds construction in the weak dissipation case, require approaches borrowed from the Hamiltonian framework.

Acknowledgments: This research was partially supported by the Italian Ministry of Education, University and Scientist Research under PRIN Grant No. 2017L7X3CS and by the Air Force Office of Scientific Research, Grant N. FA 8655-20-1-7025. The authors acknowledge the financial support from National Group of Mathematical Physics (GNFM-INdAM), from the Compagnia San Paolo, project MINIERA no. I34I20000380007 and from University of Trento, project UNMASKED 2020.

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