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Nonlinear dynamics of Disk-based MEMS Coriolis Vibrating Gyroscope under parametric excitation of vibrations

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Abstract: This work is devoted to the development of a mathematical model and a qualitative study of the nonlinear dynamics Disk-based MEMS Coriolis Vibrating Gyroscope in the free precession mode of the operating mode of oscillations with parametric excitation of the resonator. Nonlinear equations of the dynamics of a resonator on a movable base are obtained. Using asymptotic methods, the regions of parametric resonance are found, on the basis of which the amplitude-frequency characteristics of the nonlinear system are constructed.

Keywords: keyword 1, keyword 2, keyword 3 (max 5 keywords)

1. Introduction (10 point, bold)

In a gyroscope of this type, a silicon disk is connected to the armature by means of an isotropic elastic suspension, the resonator has a thickness of 40 [μm], the outer and inner radii of the disk are 420 [μm], 210 [μm], respectively, the initial data are taken for a real design from [13]. The system is considered without considering the influence of the stiffness of the elastic suspension. The natural frequency of the operating mode of the resonator is 4.53 [MHz]. The electrode structure, which has capacitive gaps of 270 [nm], consists of 24 electrodes with a circular surface, and due to the symmetry of the working mode of oscillations, it contains 4 independent groups of electrodes located symmetrically relative to the resonator [14]. The purpose of this work is to qualitatively study the dynamics of the resonator with parametric excitation of oscillations considering the geometric and electrostatic nonlinearity of the system. The tasks of the work are: creation and analysis of a compact mathematical model of MTVG, namely, the definition of zones of parametric swing oscillations; construction of resonance curves for a resonator on a fixed base in the region of the main parametric resonance and analysis of their stability.

2. Results and Discussion (10 point, bold)

On the basis of the Hamilton-Ostrogradsky variational principle and the Ritz method, a system of nonlinear differential equations is obtained that describes the dynamics of a disk resonator with an electronic control system, considering the influence of nonlinearity of geometric relationships, the presence of internal friction and the action of electrostatic forces. The equations of motion in dimensionless variables are as follows:

$$\begin{aligned}
&\ddot{C} + (1+\sigma_2)C + A_1C(C^2+S^2) + R\dot{C} + A_2\dot{C} = (A_3C + A_4C(C^2+S^2))\sin\big((2+\sigma_1)\tau\big),\\
&\ddot{S} + (1+\sigma_2)S + A_1S(C^2+S^2) + R\dot{S} - A_2\dot{S} = (A_3S + A_4S(C^2+S^2))\sin\big((2+\sigma_1)\tau\big),
\end{aligned} \tag{1}$$

where σ_2 – dimensionless detuning of the natural frequency of the system caused by the constant component of the electric field, $A_1 - A_4$ – coefficients depending on the selected operating mode of the resonator, the parameters of the material and components responsible for the electric forces $W_i, \Delta W_i$, introduced in the equation (2) , σ_1 – detuning of the excitation frequency from the frequency of the main parametric resonance for an unstressed resonator, R – friction parameter, C, S – modal coordinates normalized over the capacitive gap, τ – dimensionless time.

The voltage V_i on each group of electrodes shown in Figure 1 changes according to a harmonic law and has the form:

$$V_i^2(t) = W_i^2 + \frac{4}{\pi} W_i \Delta W_i \sin\big((2+\sigma_1)\tau\big), \; j = \overline{1,4}, \tag{2}$$

where W_i, [B]; ΔW_i, [B] – constant and variable voltage components of the i-th group of electrodes.

The asymptotic solution in the first approximation has the form:

$$\begin{gathered} C = a_1 \cos\left(\left(1 + \frac{1}{2}\sigma_1\right)t + \frac{1}{2}\psi_1\right) + O(\varepsilon), \\ S = a_2 \cos((1 + \frac{1}{2}\sigma_1)t + \frac{1}{2}\psi_2) + O(\varepsilon), \end{gathered} \tag{3}$$

where $O(\varepsilon)$ – are small expansion terms, $a_1 = a_1(t), a_2 = a_2(t)$, $\psi_1 = \psi_1(t), \psi_2 = \psi_2(t)$ – are slow amplitude-phase variables, which are found from solutions of the following system:

$$\begin{aligned} \dot{a_1} &= \frac{A_4}{8} a_1(a_1^2 + a_2^2)\cos\psi_1 - \frac{R}{2}a_1 - \frac{A_2}{2}a_2\cos\left(\frac{\psi_1}{2} - \frac{\psi_2}{2}\right) + \frac{A_3}{4}a_1\cos\psi_1 \\ &\quad + \frac{A_1}{8}a_1 a_2^2 \sin(\psi_1 - \psi_2), \\ \dot{a_2} &= \frac{A_4}{8} a_2(a_1^2 + a_2^2)\cos\psi_2 - \frac{R}{2}a_2 + \frac{A_2}{2}a_1\cos\left(\frac{\psi_1}{2} - \frac{\psi_2}{2}\right) + \frac{A_3}{4}a_2\cos\psi_1 \\ &\quad - \frac{A_1}{8}a_1^2 a_2 \sin(\psi_1 - \psi_2), \\ \dot{\psi_1} &= 2\sigma_2 - \sigma_1 + \left(\frac{3a_1^2}{4} + \frac{a_2^2}{2}\right)A_1 - \frac{A_3}{2}\sin\psi_1\left(2 + (2a_1^2 + a_2^2)\right) + \frac{A_2^2}{4} + \frac{a_2}{a_1}A_2\sin\left(\frac{\psi_1}{2} - \frac{\psi_2}{2}\right) \\ &\quad + \frac{A_4}{4}a_2^2\sin\psi_2 + \frac{A_1}{4}a_2^2\cos(\psi_1 - \psi_2), \\ \dot{\psi_2} &= 2\sigma_2 - \sigma_1 + \left(\frac{3a_2^2}{4} + \frac{a_1^2}{2}\right)A_1 - \frac{A_3}{2}\sin\psi_2\left(2 + (2a_2^2 + a_1^2)\right) + \frac{A_2^2}{4} + \frac{a_1}{a_2}A_2\sin\left(\frac{\psi_1}{2} - \frac{\psi_2}{2}\right) \\ &\quad + \frac{A_4}{4}a_1^2\sin\psi_1 + \frac{A_1}{4}a_1^2\cos(\psi_1 - \psi_2). \end{aligned} \tag{4}$$

On the basis of the developed mathematical model, the region was investigated and the region of the main parametric resonance was found. The transient curves at various values of the constant voltage are shown in Figure 1. The results obtained allowed us to estimate the starting voltages for a real structure, at a specific value of the deviation of the vibration excitation frequency (σ_1) from the natural frequency of the system $(1 + \sigma_2)$. On the basis of the obtained system (4) with the use of numerical methods of the theory of continuation, the amplitude-frequency characteristics of the resonator are constructed when the variable voltage component is varied. Figure 2 shows the dependence of the steady-state oscillation amplitude normalized with respect to the capacitive gap on the frequency detuning σ_1, the dashed line indicates the unstable branches.

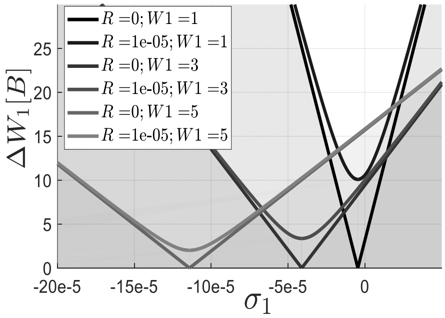

Fig. 1. Transition curves for a parametrically excited resonator, at $\Delta W_1 = \Delta W_2 = \Delta W_3 = \Delta W_4$ и $W_1 = W_2 = W_3 = W_4$.

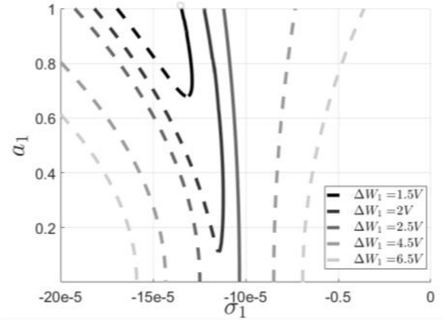

Fig.2. Amplitude-frequency characteristics for a parametrically excited resonator, constant voltage component $W_1 = W_2 = W_3 = W_4 = 5[B]$, friction parameter $R = 10^{-5}$

3. Conclusion

The paper presents a mathematical model of an CVG with a disk resonator. The nonlinear dynamics of the system is investigated in the region of the main parametric resonance. For specific parameters of the possible design of the CVG, the starting voltages were assessed. Resonance curves of the resonator are constructed, the stability of the found stationary solutions is investigated. The proposed dynamic model of CVG will be used in the future for the parametric analysis of the free precession mode of the operating mode of oscillations of the resonator on a moving base, as well as for the development of algorithms for carrying out calibration tests of the sensor in the presence of material and geometric imperfections, methods of dynamic balancing and algorithms for controlling the oscillations of the sensitive element.

References

[1] Пешехонов В. Г. Перспективы развития гироскопии //Гироскопия и навигация. – 2020. – Т. 28. – №. 2. – С. 3-10.

[2] Пешехонов В. Г. Современное состояние и перспективы развития гироскопических систем //Гироскопия и навигация. – 2011. – №. 1. – С. 3-16.

[3] Pourkamali S., Hao Z., Ayazi F. VHF single crystal silicon capacitive elliptic bulk-mode disk resonators-part II: implementation and characterization //Journal of Microelectromechanical Systems. – 2004. – Т. 13. – №. 6. – С. 1054-1062.

[4] Johari H., Ayazi F. High-frequency capacitive disk gyroscopes in (100) and (111) silicon //2007 IEEE 20th International Conference on Micro Electro Mechanical Systems (MEMS). – IEEE, 2007. – С. 47-50

[5] Mirjalili R. et al. Substrate-decoupled silicon disk resonators having degenerate gyroscopic modes with Q in excess of 1-million //2015 Transducers-2015 18th International

Conference on Solid-State Sensors, Actuators and Microsystems (TRANSDUCERS). – IEEE, 2015. – С. 15-18.

[6] Serrano D. E. et al. Substrate-decoupled, bulk-acoustic wave gyroscopes: Design and evaluation of next-generation environmentally robust devices //Microsystems & Nanoengineering. – 2016. – Т. 2. – №. 1. – С. 1-10.

[7] Lychev S. A., Manzhirov A. V., Joubert S. V. Closed solutions of boundary-value problems of coupled thermoelasticity //Mechanics of solids. – 2010. – Т. 45. – №. 4. – С. 610-623.

[8] Дурукан Я., Рыбина М. А., Шевелько М. М. Состояние и перспективы разработки чувствительных элементов на объемных акустических волнах для датчиков угловой скорости //Навигация и управление движением. – 2019. – С. 169-171.

[9] Sharma J. N., Grover D., Kaur D. Mathematical modelling and analysis of bulk waves in rotating generalized thermoelastic media with voids //Applied mathematical modelling. – 2011. – Т. 35. – №. 7. – С. 3396-3407.

[10] Jani S. M. H., Kiani Y. Generalized thermo-electro-elasticity of a piezoelectric disk using Lord-Shulman theory //Journal of Thermal Stresses. – 2020. – Т. 43. – №. 4. – С. 473-488.

[11] Yang Y. et al. Nonlinearity of degenerately doped bulk-mode silicon MEMS resonators //Journal of Microelectromechanical Systems. – 2016. – Т. 25. – №. 5. – С. 859-869.

[12] Hu Z., Gallacher B. J. Effects of Nonlinearity on the Angular Drift Error of an Electrostatic MEMS Rate Integrating Gyroscope //IEEE Sensors Journal. – 2019. – Т. 19. – №. 22. – С. 10271-10280.

[13] Serrano D. E. et al. Substrate-decoupled, bulk-acoustic wave gyroscopes: Design and evaluation of next-generation environmentally robust devices //Microsystems & Nanoengineering. – 2016. – Т. 2. – №. 1. – С. 1-10.

[14] Joubert S.V., Shatalov M.Y., Spoelstra H. (2017) On Electronically Restoring an Imperfect Vibratory Gyroscope to an Ideal State. In: Altenbach H., Goldstein R., Murashkin E. (eds) Mechanics for Materials and Technologies. Advanced Structured Materials, vol 46. Springer, Cham