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On the evolution of orbits in the circular restricted photogravitational three-body problem . Internal problem.

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Abstract: The space restricted circular three-body problem is considered in the non-resonant case. It is assumed that massless point (satellite) has a large windage so the light pressure is taken into account. We study evolution of the satellite's orbits by the Gauss method: the doubly averaged equations in the ceplerian phase space are investigated provided that the unperturbed orbit is a ceplerian ellipse with the Sun at a focus. It is also assumed that the unperturbed orbit is inside a sphere whose radius is equal to the radius of the outer planet's orbit (internal version of the three-body problem). We showed that averaged perturbing function has an explicit analytical formula expressed in terms of hypergeometric Clausen functions. By Parseval's formula and Walfram Mathematica, we got this result. It is also shown that the averaged equations of motion are integrated if the light pressure is taken into account. The first integrals are as follows: the semi-major axis of the satellite orbit, the averaged perturbing force function, the classical Lidov-Kozai integral. For the Hill case, when the values of the unperturbed semi-major axis of the satellite orbit are less than the distance to Jupiter, a study of stationary motions was carried out. Phase portraits of oscillations are constructed for different values of the light pressure coefficient δ and some fixed value of Lidov-Kozai integral.

Keywords: three-body problem, evolution of orbits

1. Introduction

The study of the orbit evolution of the planetoid Ceres in the doubly averaged circular restricted three-body problem was first carried out by C. Gauss in 1809 [1]. In the article [2], an averaged potential function of the problem was obtained, but the author used some unknown special functions in the form of quadratures. The evolving orbit topology for the internal circular three-body problem is investigated numerically in the article [3].

2. Results and Discussion

It is well known that the study of the averaged equations in the classical three-body problem faces difficulties due to the lack of an analytical formula for the averaged potential function [3]. By Parseval's formula and Walfram Mathematica, we got an explicit analytical formula for this averaged function in terms of hypergeometric Gauss, Clausen functions and Legendre polynomials:

$$R^{**} = -\frac{\delta r_0}{a(1-e^2)} + \frac{f m_J}{r_J}\sum_{n=1}^{\infty}\left[B_{2n}(e)P_{2n}(0)P_{2n}(\cos i)F_{2,1}\left(\frac{1}{2},2n;1;\frac{2e}{e-1}\right)+\right.$$
$$\left. +(-1)^n\left(\sum_{k=n}^{\infty}B_{2k}(e)A_{2n}^{(2k)}(e,\cos i)\right)\cos 2n\omega\right]$$

$$B_{2k}(e)=\frac{a^{2k}(1+e)^{2k}}{r_J^{2k}}P_{2k}(0),\quad A_{2n}^{(2k)}(e,\cos i)=2F_{3,2}^{reg}\left(\frac{1}{2},1,2k;1-2n,1+2n;\frac{2e}{e-1}\right)\times$$
$$\times\frac{(2k-n)!}{(2k+n)!}P_{2k}^{(2n)}(0)P_{2k}^{(2n)}(\cos i),\ \ if\ \ 1\le n\le k,\ \ A_{2n}^{(2k)}(e,\cos i)=0,\ \ if\ \ \ n>k$$

Here δ is the light pressure coefficient, a,e,ω,i are the semi-major axis, eccentricity, pericenter argument, tilt angle of osculating orbit, m_J, r_J are the mass of the outer planet (Jupiter) and the radius of its orbit, P_{2k}, P_{2k}^{2n} are the Legendre polynomials and associated Legendre polynomials, $F_{2,1}$ is the hypergeometric Gauss function, $F_{3,2}^{reg}$ is hypergeometric Clausen function.

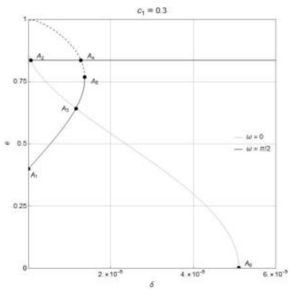
The averaged equations of motion have three first integrals in involution:

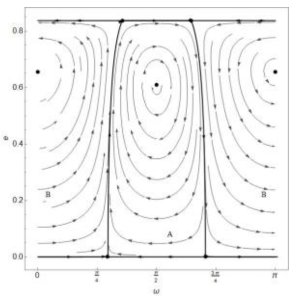
$$a=c_0,\ (1-e^2)\cos^2 i=c_1,\ R^{**}=c_2$$

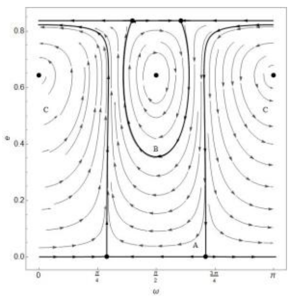
Using equations

$$\frac{dR^{**}}{de}=0,\ \frac{dR^{**}}{d\omega}=0\ ,$$

we investigated stationary motions and their bifurcations for a fixed value of c_1 when $0\le a/r_J\le 0.5$, $0<e<1$.







The first figure shows a bifurcation diagram, the second and third figures illustrate the effect of splitting separatrices.

3. Concluding Remarks

We investigated the phase portrait of oscillations in eccentricity and pericenter argument depending on the coefficient of light pressure δ, described the bifurcations of equilibria and splitting of separatrices also. We described the effect of changing the direction of pericenter argument evolution to the opposite in comparison with the classical three-body problem.

References

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