DSTA 2021
dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Performance of a nonlinear energy sink coupled with a nonlinear oscillator for energy harvesting applications

Rahul Das[1*], Anil K Bajaj[2], Sayan Gupta[1, 3]

1. Department of Applied Mechanics, Indian Institute of Technology, Madras, India. [am17d201@smail.iitm.ac.in]
2. School of Mechanical Engineering, Purdue University, Purdue, USA. [bajaj@purdue.edu]
3. Complex Systems and Dynamics Group, Indian Institute of Technology, Madras, India. [sayan@iitm.ac.in]
* Presenting Author

Abstract: The challenge in vibration based energy harvesting lies in scavenging the mechanical energy from a vibrating system at a broad range of excitation frequencies. This study focuses on using a nonlinear energy sink to increase the harvesting efficiency in nonlinear oscillators. The system consists of an oscillator with cubic nonlinearity, coupled with an essentially nonlinear cubic oscillator, suitably tuned such that the flow of energy is unidirectional towards the nonlinear attachment. This study investigates analytically, the mechanisms and conditions for unidirectional energy transfer and how the efficiency can be enhanced through proper tuning of the system parameters.

Keywords: Nonlinear energy sink, nonlinear oscillators, targeted energy transfer.

1. Introduction

The idea of simultaneously mitigating vibration and harvesting energy is very effective for various application purposes. A classical vibration based energy-harvesting device consists of a harvesting component typically made of smart materials, attached to a *primary oscillator* (*PO*) and works efficiently in a narrow band around the natural frequency of the PO. A large body of literature discussed how nonlinearities in the system can increase the operational bandwidth [1-2]. One class of such studies uses the concept of *Nonlinear Energy Sink* (*NES*), which is an essentially nonlinear oscillator, attached to a PO. The frequency energy dependency of the system due to its nonlinearity implies the possibility of the system attaining resonance condition at wide frequency bands and is exploited for energy transfer. Most of the energy of a linear PO gets transferred to the NES, in a passive, unidirectional and irreversible manner [3] and is referred to as *Targeted Energy Transfer* (*TET*). This energy transfer occurs between the substructures of the system in a passive manner. Due to frequency-energy dependency of NES, TET starts when the instantaneous frequency of NES gets close to the natural frequency of PO, attaining resonating condition. This transferred energy, increases the instantaneous frequency of NES, leading to a mistuning with the frequency of PO. As a result, the transferred energy gets localized in NES, making this entire phenomenon unidirectional. In addition, the system damping ensures that the energy transfer is irreversible.

Unlike existing studies where the PO is usually linear, in this study, the PO is assumed to be an oscillator with cubic nonlinearity, to which NES is attached. The motivation for selecting such a system is to investigate vibration-based energy harvesting from physical systems, which are usually flexible and therefore involve nonlinear effects due to large deformations or fluid-structure interaction effects. To get insights into the physics of TET, an analytical approach is adopted. This involves adopting the method of *complexification-averaging* (*CX-A*) [4] for studying the system across multiple time scales such that the conditions of 1:1 resonance capture between the PO and NES can be derived. This enables identifying the optimal TET regime. The system parameters are selected by adopting a tuning methodology [5], which emphasises the optimal TET condition. The modal response of the system at resonance condition are simulated using a harmonic balance based approach, followed by arc-length type continuation process.

2. Results and Discussion

The different TET regimes for the system comprising of a Duffing oscillator attached with a NES is shown in Fig. 1 with respect to the variation of the magnitude of the relative amplitude of the complexified-averaged system, $|u(t)|$ and time. This is a measure of energy transfer as transferred energy is directly related to the relative motion of the substructures. It is observed that there exists three distinct regimes. The initial regime comprises of a segment where energy is transferred due to nonlinear beating phenomenon near $S11^{+}$-the in-phase 1:1 resonating manifold. This is followed by an intermediate regime, govened by damping. Finally, there is a transition towards $S11^{-}$-the out-of-phase 1:1 resonating manifold, where the energy is completely localized in the NES. More results on finding the dynamics at different regimes will be presented in the work.

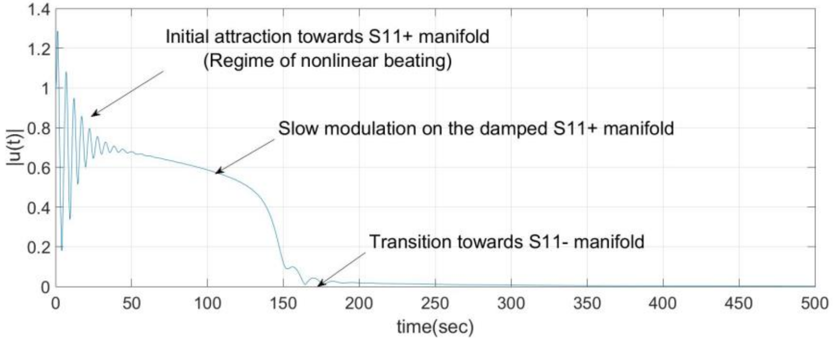

Fig. 1. Different TET regimes on the slow modulation of relative amplitude of complexified-averaged system.

References

[1] D. D. Quinn, O. V. Gendelman, G. Kerschen, T. P. Sapsis, L. A. Bergman, A. F. Vakakis, Efficiency of targeted energy transfers in coupled nonlinear oscillators associated with 1:1 resonance captures: part I, Journal of sound and Vibration 311(3-5) (2008) 1228-1248.

[2] T. P. Sapsis, A. F. Vakakis, O. V. Gendelman, L. A. Bergman, G. Kerschen, D. D. Quinn, Efficiency of targeted energy transfers in coupled nonlinear oscillators associated with 1:1 resonance captures: part II, Journal of sound and Vibration 325(1-2) (2009) 297-320.

[3] A. F. Vakakis, O. V. Gendelman, L. A. Bergman, D. M. McFarland, G. Kerschen, Y. S. Lee, Nonlinear targeted energy transfer in mechanical and structural systems, Vol. 156, Springer Science & Business Media, 2008.

[4] L. I. Manevitch, Complex representation of dynamics of coupled nonlinear oscillators, In Mathematical models of nonlinear excitations, transfer, dynamics, and control in condensed systems and other media, Springer, Boston, MA, (1999) 269-300.

[5] R. Vigui´e, G. Kerschen, Nonlinear vibration absorber coupled to a nonlinear primary system: a tuning methodology, Journal of sound and Vibration 326 (3-5) (2009) 780–793.