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Modelling and analysing of a spring pendulum motion in the presence of energy harvesting devices

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Abstract: Energy harvesting will become more and more essential mechanical vibration applications of many devices. The vibrations can be converted by appropriate devices into electrical energy, which can be used as a power supply instead of ordinary ones. This paper investigates a dynamical system associated with two devices: a piezoelectric device and an electromagnetic one. These devices are connected with a nonlinear damping spring pendulum with 2DOF, in which its supported point moves in a circular path. The equations of motion are obtained using Lagrange's equations of the second kind. The asymptotic solutions of these equations are obtained up to the third approximation utilizing the perturbation approach of multiple scales. The comparison between these solutions and the numerical ones reveals high consistency between them. The steady-state solutions are obtained, and their stabilities are tested. The influences of the excitation amplitudes, the damping coefficients, and the different frequencies on energy harvesting devices outputs are examined and discussed. The work is essential due to its significance in real-life applications. The developed methodology and obtained results can be useful in various applications like power supply of sensors and charging electronic devices.

Keywords: Perturbation methods, Resonance, Stability.

1. Introduction

Energy harvesting (EH) is an essential vital aspect that the research fields work on through the previous few years. It transforms the surrounding energy present in the Earth into electrical power to drive autonomous electronic devices or circuits [1-2]. This energy can be harvested from solar energy, thermal energy, and the most crucial source for harvesting is kinetic energy, especially from vibrational motion [3].

2. Results and Discussion

Two energy harvesting devices, including piezoelectric and electromagnetic, are connected with a pendulum separately 'as two cases' to convert the vibrational motion into electrical. The main governing system of motion is derived using Lagrange's equations, in which the mechanism of the piezoelectric and electromagnetic circuits are used to obtain their corresponding equations as follows

$$\ddot{z}+c_1\dot{z}+z+3\alpha\zeta_s{}^2z+3\alpha\zeta_s z^2+\alpha z^3-r\,p^2(\cos p\tau+\phi\sin p\tau)+\frac{1}{2}W^2\phi^2+PM-(1+z)\dot{\phi}^2=f_1\cos p_1\tau,$$

$$(1+z)^2\ddot{\phi}+c_2\dot{\phi}+W^2(1+z)(\phi-\frac{\phi^3}{6})-r\,p^2(1+z)(\sin p\tau-\phi\cos p\tau)+2(1+z)\dot{z}\dot{\phi}=f_2\cos p_2\tau,$$

$$\dot{v}+\frac{v}{R_p c_p \omega_1}=\frac{l\gamma_1}{c_p}\dot{z},\qquad \dot{q}+\frac{q\,R_m}{l_m\omega_1}=\frac{l\gamma_2}{l_m}\dot{z}.$$

The asymptotic solutions up the third approximation are obtained utilizing the multiple scale method. External resonance cases between the classified cases of resonance are examined. The numerical results are compared with the approximate ones to reveal the accuracy between them (Fig.1). The modulation equations are obtained to explore the solutions at the steady-state and to examine the stability of the fixed points. The influence of damping coefficients and excitation amplitudes on the output voltage, current, and power are represented graphically (see Fig.2).

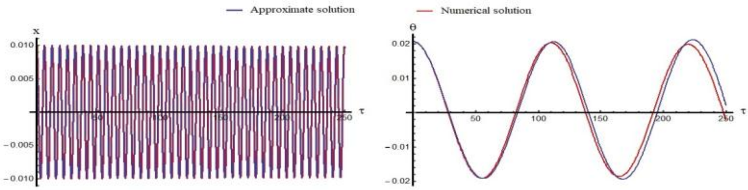

Fig.1: The comparison between the numerical and the approximate solutions.

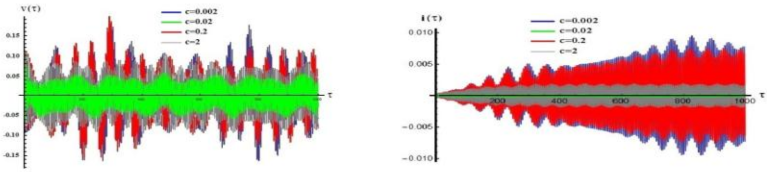

Fig.2: Time histories for the voltage (V) from the piezoelectric device and the current (i) from the electromagnetic device with different values of damping coefficient.

3. Concluding Remarks

A nonlinear damped vibrating spring pendulum system with 2-DOF moving in a circular path with constant angular velocity is investigated which is connected with energy harvesting devices. The governing equations are obtained using Lagrange's equations and solved using the multiple scale method. The time histories of dynamical motion, the responses of the resonance curves, and the solutions at the steady-state cases are plotted to reveal the good impact of the selected values of the model parameters on the motion. A comparison between the numerical and approximate solutions shows high consistency between them. The influence of the damping coefficients and the excitation amplitude on the output voltage, current, and power is examined. Moreover, the response of excitation frequency on the output power of the systems is checked. Electrical energy was generated from the piezoelectric and electromagnetic devices, which were attached to the vibrating system.

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