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Energy exchanges in a nonlinear meta-cell

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Abstract: Vibratory Energy exchanges between particles of a nonlinear meta-cell are studied. The meta-cell is composed of an inner mass with a combined nonlinear restoring forcing term covered by an outer mass. A time multiple scales analysis is carried out in order to find the slow invariant manifold (SIM) and also singular and equilibrium points of the system. A combined nonlinear restoring forcing function of the inner mass makes the behaviour of the SIM different from classical corresponding ones. Finally, quasi-analytical system responses are confronted with numerical ones, obtained by direct integration.

Keywords: meta-cell, nonlinear restoring force, slow invariant manifold, equilibrium/singular points

1. Introduction

In contrast to the classical linear vibration absorber, a nonlinear energy sink (NES) can resonantly interact with the main system for wide frequency ranges [1]. The first studies concerning the NES were based on cubic nonlinearity [2]. Gendelman [3] studied NES with non-polynomial restoring forcing terms, while Lamarque et al [4] investigated systems with piece-wise linear restoring forcing terms. Vibro-impact NES were studied by Gendelman [5] and Gourc et al [6]. In this paper, the response of a single meta-cell with two degrees-of-freedom and a combined nonlinear restoring forcing term is investigated. The organization of the article is as it follows: The analytical methods applied for solving the equations of motion are discussed in Sect. 2. A numerical example is presented in Sect. 3. Finally, the paper is concluded in Sect. 4.

2. Detection of dynamical characteristic of the system

The governing system equations of the 2-dof meta-cell motion have the following expression:

$$\begin{cases} m_1\ddot{u}_1 + K_1 u_1 + C_1\dot{u}_1 + F(u_1 - u_2) + C_2(\dot{u}_1 - \dot{u}_2) = P\sin(\omega t) \\ m_2\ddot{u}_2 + F(u_2 - u_1) + C_2(\dot{u}_1 - \dot{u}_2) = 0 \end{cases}, \quad (1)$$

with m_1 and m_2 the outer and inner masses, K_1 and C_1 the linear stiffness and the damping coefficients of the outer mass, C_2- the damping coefficient of the inner mass. Furthermore, $F(\alpha)$ is an odd restoring forcing function of the inner mass. In this study, a combination of a cubic and a non-smooth nonlinearity as $F(\alpha)$ is considered. A nondimensionalized time $\tau = t\sqrt{K/m}$, the coordinates of relative displacement and the centre of mass are introduced to the system variables. After this, the complex variables of Manevitch [7] are applied. A Galerkin method based on truncated Fourier series, via keeping the first harmonics of the system, is employed [8] and a time multiple scales analysis [9] is carried out in order to find the SIM and singular and equilibrium points of the system.

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3. Numerical example

The quasi-analytical results for a free system are confronted with those obtained by the direct numerical integration of equations for two different initial conditions. Results are depicted in Fig. 1. The numerical results follow the SIM of the system which possesses two pairs of local extrema (due to the especial nonlinearity of the system) and after one or two bifurcations (depending on the initial conditions), reaches to the rest position.

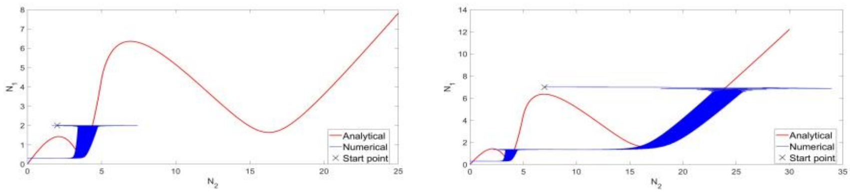

Fig. 1. The SIM and corresponding numerical results for two different initial conditions: ($N_{1,}$ N_2) = (2,2) and ($N_{1,}$ N_2) = (7,7). N_1 and N_2 stand for amplitudes of the outer and inner masses, respectively.

4. Concluding Remarks

The results presented show that a meta-cell with combined nonlinearity may exhibit a slow invariant manifold which is different from classical systems [3], leading to energy exchanges between particles via several bifurcations.

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