16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Dynamics of a low-inertia ball located between two rotating planes with viscous friction

ALEXANDER KOSHELEV, EUGENE KUGUSHEV, TATIANA SHAHOVA*

Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow, Russia

Abstract: The problem of the motion of a low-inertia ball between two horizontal uniformly rotating planes with linear viscous friction is considered. The center of mass of the ball coincides with its geometric center, the central inertia tensor of the ball is spherical. Two cases of low inertia of the ball are investigated: in the first case the mass of the ball is constant and concentrated near its center, in the second case the mass of the ball is small. The dynamics of the ball on an arbitrary finite time interval in the limit as the central moment of inertia of the ball tends to zero is studied. In both cases the equations of motion of the ball have the form of the Tikhonov's type equations with a small parameter in the left-hand side. In the first case it is shown that in the limit the center of mass of the ball moves as a material point located between horizontal rotating planes with linear viscous friction. In the second case the dynamics of the ball in the limit coincides with the dynamics of a homogeneous ball moving between two absolutely rough rotating planes.

Keywords: low-inertia ball, viscous friction, small parameter, Tikhonov's theorem.

1. Introduction

Non-ideality of hinges can significantly affects on dynamics of mechanical systems containing hinges. One of the reasons for the violation of the ideality of the hinge is the presence of small alien bodies between the working surfaces of the hinge. The problem of the migration of such alien bodies with relative movements of the working surfaces of the hinge is interesting. In the simplest case the dynamics of a ball constrained by two parallel rotating planes with linear viscous friction is researched. The dynamics of a ball along a rotating plane with linear viscous friction was studied in [1].

2. Description of the mechanical system and main result

Suppose a ball of mass *m* and radius *a* moves between two horizontal planes. Each plane rotates with the constant angular velocity of value Ω_i around some fixed vertical axis. The force of linear viscous friction $\mathbf{F}_i = -c_i \mathbf{u}_i$ acts on the point of contact of the ball with the plane, where c_i is the coefficient of viscous friction, $\mathbf{u}_i$ is the velocity of the contact point relative to the rotating plane ($i = 1, 2$). The distance between the axes of rotation of the planes is equal to $2l$. The center of mass of the ball coincides with its geometric center, the central inertia tensor of the ball is spherical, the principal central moment of inertia of the ball is equal to *I*. Let us research the motion of the ball as *I* tends to zero.

Let us introduce a fixed coordinate system *Oxyz* so that *Oxy* is the plane of motion of the ball center, *Oz* axis is vertical, *Oy* axis crosses the rotation axes of the planes and the point *O* is equidistant from these axes. The equations of motion of the ball have the form

* 119991, Russia, Moscow, GSP-1, Leninskie Gory 1, Main Building; e-mail: t.shahova@yandex.ru

$$\frac{d\mathbf{r}}{dt} = \mathbf{v},\ m\frac{d\mathbf{v}}{dt} = -\alpha\mathbf{v} + [\mathbf{e}_z, -\beta a\boldsymbol{\omega}_{\|} + \gamma\mathbf{r} + \delta\mathbf{l}],\ I\frac{d\boldsymbol{\omega}_{\|}}{dt} = \beta a[\mathbf{e}_z, \mathbf{v}] - \alpha a^2\boldsymbol{\omega}_{\|} + \delta a\mathbf{r} + \gamma a\mathbf{l},\ \frac{d\omega_z}{dt} = 0. \quad (1)$$

Here $\mathbf{r}$, $\mathbf{v}$ are the radius vector and the velocity of the center of mass of the ball, $\boldsymbol{\omega} = \boldsymbol{\omega}_{\|} + \omega_z\mathbf{e}_z$ is the angular velocity of the ball, $\mathbf{l} = l\mathbf{e}_y$, $\alpha = c_1 + c_2$, $\beta = c_1 - c_2$, $\gamma = c_1\Omega_1 + c_2\Omega_2$, $\delta = c_1\Omega_1 - c_2\Omega_2$.

Let the mass of the ball be constant and concentrated near its center. Then $I = mr^2$, where $r = \sqrt{\varepsilon}a$ ($0 < \varepsilon << 1$) is the radius of inertia of the ball relative to an arbitrary axis passing through the center of mass of the ball. In this case the third group of equations (1) has small parameter ε in the left side. Using Tikhonov's theorem [2] the statement is proved.

Statement 1. Let $\mathbf{r}(t,\varepsilon), \mathbf{v}(t,\varepsilon), \boldsymbol{\omega}(t,\varepsilon)$ be the solution of equations (1) with initial conditions $\mathbf{r}(0) = \mathbf{r}_0, \mathbf{v}(0) = \mathbf{v}_0, \boldsymbol{\omega}(0) = \boldsymbol{\omega}_0$ on some finite time interval $t \in [0,T]$ for any $\varepsilon \in (0,\varepsilon_0]$. Then as $\varepsilon \to 0$ the functions $\mathbf{r}(t,\varepsilon), \mathbf{v}(t,\varepsilon)$ converge on the interval $t \in [0,T]$ to the solution $\mathbf{r}^*(t), \mathbf{v}^*(t)$ of

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad m\frac{d\mathbf{v}}{dt} = \frac{2c_1c_2}{c_1 + c_2}\left(-2\mathbf{v} + [\mathbf{e}_z, (\Omega_1 + \Omega_2)\mathbf{r} + (\Omega_1 - \Omega_2)\mathbf{l}]\right) \quad (2)$$

with the initial conditions $\mathbf{r}^*(0) = \mathbf{r}_0, \mathbf{v}^*(0) = \mathbf{v}_0$ and the function $\boldsymbol{\omega}(t,\varepsilon)$ converges on the interval $t \in (0,T]$ to the function

$$\boldsymbol{\omega}^*(t) = \frac{1}{(c_1 + c_2)a}\left((c_1 - c_2)[\mathbf{e}_z, \mathbf{v}^*(t)] + (c_1\Omega_1 - c_2\Omega_2)\mathbf{r}^*(t) + (c_1\Omega_1 + c_2\Omega_2)\mathbf{l}\right) + (\boldsymbol{\omega}_0, \mathbf{e}_z)\mathbf{e}_z .$$

Equations (2) have the form of equations of motion of a material point located between two horizontal uniformly rotating planes with linear viscous friction.

Let the mass of the ball be small. Then $I = mr^2$, where $m = \varepsilon M$ ($0 < \varepsilon << 1$), M is the characteristic mass. The second and third groups of equations (1) have small parameter ε in the left side.

Statement 2. Let $\mathbf{r}(t,\varepsilon), \mathbf{v}(t,\varepsilon), \boldsymbol{\omega}(t,\varepsilon)$ be the solution of equations (1) with initial conditions $\mathbf{r}(0) = \mathbf{r}_0, \mathbf{v}(0) = \mathbf{v}_0, \boldsymbol{\omega}(0) = \boldsymbol{\omega}_0$ on some finite time interval $t \in [0,T]$ for any $\varepsilon \in (0,\varepsilon_0]$. Then as $\varepsilon \to 0$ the function $\mathbf{r}(t,\varepsilon)$ converges on the interval $t \in [0,T]$ to the solution $\mathbf{r}^*(t)$ of the equation $\frac{d\mathbf{r}}{dt} = \frac{1}{2}[\mathbf{e}_z, (\Omega_1 + \Omega_2)\mathbf{r} + (\Omega_1 - \Omega_2)\mathbf{l}]$ with the initial conditions $\mathbf{r}^*(0) = \mathbf{r}_0$ and the functions $\mathbf{v}(t,\varepsilon), \boldsymbol{\omega}(t,\varepsilon)$ converge on the interval $t \in (0,T]$ to the functions

$$\mathbf{v}^*(t) = \frac{1}{2}[\mathbf{e}_z, (\Omega_1 + \Omega_2)\mathbf{r}^*(t) + (\Omega_1 - \Omega_2)\mathbf{l}], \quad \boldsymbol{\omega}^*(t) = \frac{1}{2a}\left((\Omega_1 - \Omega_2)\mathbf{r}^*(t) + (\Omega_1 + \Omega_2)\mathbf{l}\right) + (\boldsymbol{\omega}_0, \mathbf{e}_z)\mathbf{e}_z .$$

In this case the motion of the ball in the limit coincides with the motion of a homogeneous ball located between two absolutely rough rotating planes.

3. Conclusion

The dynamics of a ball moving between two rotating planes with linear viscous friction on some finite time interval in the limit as the central moment of inertia of the ball tends to zero has been researched.

References

[1] IVANOVA T: The Rolling of a Homogeneous Ball with Slipping on a Horizontal Rotating Plane. *Russian Journal of Nonlinear Dynamics* 2019, **15**(2):171-178.

[2] TIKHONOV A: Systems of differential equations containing small parameters in the derivatives. *Matematicheskii Sbornik* 1952, **31**(3):575-586.