

## Critical tolerance evolution: Classification of the chain-recurrent set

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*Abstract:* Complete Lyapunov functions for non-linear dynamical systems can be obtained by approximately solving a partial differential equation that describes the condition for its orbital derivative. Efficient algorithms to compute them have been implemented. The fact that the partial differential equation is not satisfied at points of the chain-recurrent set is used to determine this set; more precisely, all points where the value of the orbital derivative is larger than a fixed, critical tolerance parameter, are an estimate of the chain-recurrent set. The mathematical conditions of smoothness over the orbital derivative are obtained by averaging the values of the orbital derivative locally. Furthermore, convergence to zero is avoided by normalizing the sum of the orbital derivative condition. However, the tolerance parameter to describe the chain-recurrent set has not been considered. This results in an overestimation of the chain-recurrent set. Several algorithms have been proposed to reduce the overestimation of the chain-recurrent set, but no systematic analysis on the dependence on the critical parameter has been made so far. In this paper, we focus on studying this parameter. To proceed, the chain-recurrent set is divided into different subsets of connected components; their evolution per iteration and their different behaviour are studied. The outcome of this research will create an efficient analysis method for the chain-recurrent set and aims to reduce the overestimation by obtaining the lowest possible tolerance parameter necessary to classify the chain-recurrent set.

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