

Numerical procedure for sensitivity analysis of hybrid systems

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Abstract: The paper presents the numerical procedure for the evaluation of adjoint equations for hybrid systems i.e. the systems with mixed discrete-continuous dynamics. With the help of adjoint equations the gradients of state functionals with respect to controls are calculated efficiently. Our procedure can be used in optimization procedures for solving optimal control problems with hybrid systems. The procedure is based on the implementation of a Runge-Kutta method which is advocated as the most suitable numerical procedure for integration of differential equations with controls represented by piecewise constant controls ([Pytlak, Numerical Methods for Optimal Control Problem With State Constraints, Lecture Notes in Mathematics, Springer-Verlag, 1999], [Hager, Runge-Kutta methods in optimal control and the transformed adjoint equations, Numer. Math., 2000]). Since we are dealing with hybrid systems our numerical procedure is equipped with the procedure for locating switching points which determine the change of a discrete state of the hybrid system. The evaluation of adjoint equations is consistent with the system equation discretization. We show that discrete time adjoint equations for the discretized system equations resulting from applying Radau IIa integration scheme are in fact Radau Ia integration scheme applied to the continuous time adjoint equations. We show the effectiveness of our procedure on several examples such as the Coulomb-Striebeck friction model and the model of a racing car motion with drift.

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