

Analytical and numerical study of piecewise linear Mathieu equation with non-zero offset

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Abstract: The current work considers the analytical and numerical study of piecewise linear Mathieu equation with non-zero offset. The considered parametrically excited dynamical system is such that the stiffness coefficient is unity for $q < a$ and $\delta \geq 1$ for $q \geq a$, where δ is the asymmetry parameter and a is the offset at which the system exhibits asymmetry. The system under consideration is essentially nonlinear and as such, not linearizable. Thus the application of Floquet theory is seldom possible. The previous work by Chatterjee et al. ("Asymmetric Mathieu Equation," Proc. of the Royal Soc. A, vol. 462, p. 1643-1659, 2006) introduced a numerical measure to ascertain the stability of the oscillator response in the limit of zero offset. This was rendered possible due to the scalable property of the system with zero offset and in fact the system exhibits isochronous oscillations. In contrast, the presence of non-zero offset precludes the applicability of such a measure and the general quasi-linear asymptotic analysis is seldom applicable. Furthermore, the system under consideration exhibits non-isochronous oscillations and the frequency of oscillation approaches an asymptotic limit (frequency of oscillation with zero offset) with increase in energy. In this study we invoke the canonical variable of action-angle (AA) and consider appropriate averaging to project the dynamics on a resonance manifold. The averaged equations provide a measure of the instability boundaries in this class of systems. The numerical simulations are observed to match quite well with the asymptotic solutions.

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